a) erforderlicher Passfugendruck

$$p_{erf} := \frac{v_{Rmin}}{2\ \pi \cdot r_p \cdot l_f \cdot \mu_H} \cdot \sqrt{{F_{ax}}^2 + \left(\frac{T}{r_p}\right)^2}$$

$$p_{erf} := \frac{2}{2\ \pi \cdot 25\ mm \cdot 40\ mm \cdot 0.2} \cdot \sqrt{(4000\ N)^2 + \left(\frac{300000\ N \cdot mm}{25\ mm}\right)^2} = 20.132\ \frac{N}{mm^2}$$

b) Übermaßverlust

$$G := 0.4\ (R_{z.Welle} + R_{z.Nabe}) \qquad G := 0.4\ (3\ \mu m + 3\ \mu m) = 2.4\ \mu m$$

c) zulässige Passfugendurck

$$q_{Nabe} := \left(\frac{r_{i.Nabe}}{r_{a.Nabe}}\right)^2 \qquad q_{Nabe} := \left(\frac{25\ mm}{50\ mm}\right)^2 = 0.25$$

$$q_{Welle} := \left(\frac{r_{i.Welle}}{r_{a.Welle}}\right)^2 \qquad q_{Welle} := \left(\frac{0\ mm}{25\ mm}\right)^2 = 0$$

$$\sigma_{zul} := \frac{R_{p0.2}}{v_F} \qquad \sigma_{zul} := \frac{450\ \frac{N}{mm^2}}{1.3} = 346.154\ \frac{N}{mm^2}$$

$$p_{zul} := \sigma_{zul} \cdot \frac{1 - q_{Nabe}}{2} \qquad p_{zul} := 346.154\ \frac{N}{mm^2} \cdot \frac{1 - 0.25}{2} = 129.808\ \frac{N}{mm^2}$$

d) erforderliches und zulässiges Übermaß

$$S_{erf} := p_{erf} \cdot r_p \left[\frac{1}{E_{Nabe}} \left(\left(\frac{1 + q_{Nabe}}{1 - q_{Nabe}}\right) + \nu_{Nabe}\right) + \frac{1}{E_{Welle}} \left(\left(\frac{1 + q_{Welle}}{1 - q_{Welle}}\right) - \nu_{Welle}\right)\right] + G$$

$$\frac{1 + q_{Nabe}}{1 - q_{Nabe}} = \frac{1 + 0.25}{1 - 0.25} = 1.667 \qquad \frac{1 + q_{Welle}}{1 - q_{Welle}} = \frac{1 + 0}{1 - 0} = 1$$

$$E_{Nabe} = \quad E_{Welle} := 210000\ \frac{N}{mm^2}$$

$$S_{erf} := p_{erf} \cdot r_p\ \frac{1}{E_{Nabe}} \left(\left(\frac{1 + q_{Nabe}}{1 - q_{Nabe}}\right) + \nu_{Nabe} + \left(\frac{1 + q_{Welle}}{1 - q_{Welle}}\right) - \nu_{Welle}\right) + G$$

$$S_{erf} := 20.132\ \frac{N}{mm^2} \cdot 25\ mm \cdot \frac{1}{210000\ \frac{N}{mm^2}} ((1.667 + 0.3) + (1 - 0.3)) + 2.4\ \mu m$$

$$S_{erf} = 8.792\ \mu m$$

$$S_{zul} := p_{zul} \cdot r_p \frac{1}{E_{Nabe}} \left(\left(\frac{1+q_{Nabe}}{1-q_{Nabe}} \right) + \nu_{Nabe} + \left(\frac{1+q_{Welle}}{1-q_{Welle}} \right) - \nu_{Welle} \right)$$

$$S_{zul} := 129.808 \frac{N}{mm^2} \cdot 25\ mm \cdot \frac{1}{210000 \frac{N}{mm^2}} \left((1.667+0.3) + (1-0.3) \right)$$

$$S_{zul} = 41.214\ \mu m$$

e) kleinste und größte Übermaß mit ausgewählter Passung

50H6: $EI := 0\ \mu m$ $ES := 16\ \mu m$
50s6: $ei := 43\ \mu m$ $es := 59\ \mu m$

$$S_{min} := \frac{ei - ES}{2} - G \qquad S_{min} := \frac{43\ \mu m - 16\ \mu m}{2} - 2.4\ \mu m = 11.1\ \mu m$$

$$S_{max} := \frac{es - EI}{2} \qquad S_{max} := \frac{59\ \mu m - 0\ \mu m}{2} = 29.5\ \mu m$$

f) übertragbare Kraft an der Passfugenfläche, Rutschsicherheit

$$p_{min} := \frac{S_{min}}{r_p} \cdot \frac{1}{\frac{1}{E_{Nabe}} \left(\frac{1+q_{Nabe}}{1-q_{Nabe}} + \nu_{Nabe} \right) + \frac{1}{E_{Welle}} \left(\frac{1+q_{Welle}}{1-q_{Welle}} - \nu_{Welle} \right)}$$

$$p_{min} := \frac{S_{min}}{r_p} \cdot E \cdot \frac{1}{\frac{1+q_{Nabe}}{1-q_{Nabe}} + \frac{1+q_{Welle}}{1-q_{Welle}}}$$

$$p_{min} := \frac{11.1\ \mu m}{25\ mm} \cdot 210000 \frac{N}{mm^2} \cdot \frac{1}{\frac{1+0.25}{1-0.25} + \frac{1+0}{1-0}} = 34.965 \frac{N}{mm^2}$$

$$F_{r※} := \mu_H \cdot p_{min} \cdot 2\pi \cdot r_p \cdot l_f \qquad F_{r※} := 0.2 \cdot 34.965 \frac{N}{mm^2} \cdot 2\pi \cdot 25\ mm \cdot 40\ mm$$

$$F_{r※} = 43.938\ kN$$

$$F_r := \sqrt{F_{ax}^2 + \left(\frac{T}{r_p} \right)^2} \qquad F_r := \sqrt{(4000\ N)^2 + \left(\frac{300000\ N \cdot mm}{25\ mm} \right)^2} = 12.649\ kN$$

$$v_R := \frac{F_{r※}}{F_r} \qquad v_R := \frac{43.938\ kN}{12.649\ kN} = 3.474 \qquad v_R > v_{Rmin}$$

g) maximaler Passfugendruck

$$p_{max} := \frac{S_{max}}{r_p} \cdot \frac{1}{\frac{1}{E_{Nabe}}\left(\frac{1+q_{Nabe}}{1-q_{Nabe}}+\nu_{Nabe}\right)+\frac{1}{E_{Welle}}\left(\frac{1+q_{Welle}}{1-q_{Welle}}-\nu_{Welle}\right)}$$

$$p_{max} := \frac{S_{max}}{r_p} \cdot E \cdot \frac{1}{\frac{1+q_{Nabe}}{1-q_{Nabe}}+\frac{1+q_{Welle}}{1-q_{Welle}}}$$

$$p_{max} := \frac{29.5\ \mu m}{25\ mm} \cdot 210000\ \frac{N}{mm^2} \cdot \frac{1}{\frac{1+0.25}{1-0.25}+\frac{1+0}{1-0}} = 92.925\ \frac{N}{mm^2}$$

h) Nötige Fügetemperatur (mit Schrumpfübermaß)

$$\nu_F := \frac{S_{max}+S_{ü}}{r_p \cdot \beta} + \nu_{Raum} \qquad S_{ü} := 0.001 \cdot r_p \qquad S_{ü} := 0.001 \cdot 25\ mm = 25\ \mu m$$

$$\nu_F := \frac{29.5\ \mu m + 25\ \mu m}{25\ mm \cdot 12 \cdot \left(10^{-6}\right) \cdot \frac{1}{\Delta °C}} + 20\ °C = 201.667\ °C$$

i) Festigkeitsnachweis an kritischer Stelle: für Nabe

$$\sigma_{riN} := -p_{max} = -92.925\ \frac{N}{mm^2}$$

$$\sigma_{raN} := 0\ \frac{N}{mm^2}$$

$$\sigma_{\varphi iN} := \frac{1+q_{Nabe}}{1-q_{Nabe}} \cdot p_{max} \qquad \sigma_{\varphi iN} := \frac{1+0.25}{1-0.25} \cdot 92.925\ \frac{N}{mm^2} = 154.875\ \frac{N}{mm^2}$$

$$\sigma_{\varphi aN} := \frac{2 \cdot q_{Nabe}}{1-q_{Nabe}} \cdot p_{max} \qquad \sigma_{\varphi aN} := \frac{2 \cdot 0.25}{1-0.25} \cdot 92.925\ \frac{N}{mm^2} = 61.95\ \frac{N}{mm^2}$$

$$\sigma_V := \sqrt{\sigma_{\varphi iN}^2 + \sigma_{riN}^2 - \sigma_{\varphi iN} \cdot \sigma_{riN}}$$

$$\sigma_V := \sqrt{\left(154.875\ \frac{N}{mm^2}\right)^2 + \left(-92.925\ \frac{N}{mm^2}\right)^2 - \left(154.875\ \frac{N}{mm^2}\right) \cdot \left(-92.925\ \frac{N}{mm^2}\right)}$$

$$\sigma_V = 216.825\ \frac{N}{mm^2}$$

Aufgabe 2

Deklarationen

$\alpha := 60\ °$

$\alpha_A := 1.6$

$A_{ers} := 255 \cdot mm^2$

$A_D := 7000\ mm^2$

$d := 8\ mm$

$d_h := 9\ mm$

$d_w := 13\ mm$

$d_2 := 7.19\ mm$

$d_3 := 6.47\ mm$

$E_{gg} := 110\ \frac{kN}{mm^2}$

$E_s := 210\ \frac{kN}{mm^2}$

$F_{Kges} := 4\ kN$

$f_{zs} := 4\ \mu m$

$f_{zp} := 4\ \mu m$

$i := 4$

$l_g := 10\ mm$

$l_k := 32\ mm$

$l_s := 22\ mm$

$\mu := 0.16$

$n := 1$

$p := 1.25\ mm$

$P_i := 2 \cdot MPa$

$R := 100\ mm$

$R_i := 70\ mm$

$R_{m.D} := 75\ mm$

$R_{0.2} := 900\ MPa$

Aufgabe

a)

$A_D := R_i^2 \cdot \pi = \left(1.539 \cdot 10^4\right)\ mm^2$

$R_i = 70\ mm$

$i = 4$

$F_A := P_i \cdot \frac{A_D}{i} = 7.697\ kN$

$P_i = 2\ MPa$

$A_D = \left(1.539 \cdot 10^4\right)\ mm^2$

b)

$\delta_{SK} := \frac{0.4 \cdot d}{E_s \cdot \pi \cdot \left(\frac{d}{2}\right)^2} = \left(3.032 \cdot 10^{-7}\right)\ \frac{mm}{N}$

$d = 8\ mm$ $\quad E_s = 210\ GPa$

$\delta_{gew} := \frac{l_g}{E_s \cdot \pi \cdot \left(\frac{d_3}{2}\right)^2} = \left(1.448 \cdot 10^{-6}\right)\ \frac{mm}{N}$

$l_g = 10\ mm$ $\quad E_s = 210\ GPa$

$d_3 = 6.47\ mm$

$\delta_l := \frac{l_s}{E_s \cdot \pi \cdot \left(\frac{d}{2}\right)^2} = \left(2.084 \cdot 10^{-6}\right)\ \frac{mm}{N}$

$l_s = 22\ mm$ $\quad E_s = 210\ GPa$

$d = 8\ mm$

$\delta_{GM} := \frac{0.4 \cdot d}{E_s \cdot \pi \cdot \left(\frac{d}{2}\right)^2} = \left(3.032 \cdot 10^{-7}\right)\ \frac{mm}{N}$

$d = 8\ mm$ $\quad E_s = 210\ GPa$

$\delta_s := \delta_{SK} + \delta_l + \delta_{gew} + \delta_{GM} = \left(4.139 \cdot 10^{-6}\right)\ \frac{mm}{N}$

$\delta_f := \frac{l_k}{E_{gg} \cdot A_{ers}} = \left(1.141 \cdot 10^{-6}\right)\ \frac{mm}{N}$

$l_k = 32\ mm$ $\quad A_{ers} = 255\ mm^2$

$E_{gg} = 110\ GPa$

$$\phi := \frac{\delta_f}{\delta_f + \delta_s} = 0.216$$

$$\delta_f = \left(1.141 \cdot 10^{-6}\right) \frac{mm}{N}$$

$$\delta_s = \left(4.139 \cdot 10^{-6}\right) \frac{mm}{N}$$

c)

$$f_z := f_{zs} + f_{zp} = 8 \ \mu m$$

$$F_{sz} := f_z \cdot \frac{\phi}{\delta_f} = 1.515 \ kN$$

$\phi = 0.216$ $\quad \delta_f = \left(1.141 \cdot 10^{-6}\right) \frac{mm}{N}$

d)

$F_{Kerf} := \frac{F_{Kges}}{4} = 1 \ kN$ $\quad F_{Kges} = 4 \ kN$

$$F_{Mmin} := F_{Kerf} + (1 - \phi) \cdot F_A + F_{sz} = 8.549 \ kN$$

$F_{Kerf} = 1 \ kN$ $\quad \phi = 0.216$ $\quad F_A = 7.697 \ kN$ $\quad F_{sz} = 1.515 \ kN$

$$F_{Mmax} := F_{Mmin} \cdot \alpha_A = 13.678 \ kN$$

$F_{Mmin} = 8.549 \ kN$ $\quad \alpha_A = 1.6$

e)

$$f_s := F_{Mmin} \cdot \delta_s = 35.383 \ \mu m$$

$$f_f := F_{Mmin} \cdot \delta_f = 9.753 \ \mu m$$

$F_{Mmin} = 8.549 \ kN$

$\delta_s = \left(4.139 \cdot 10^{-6}\right) \frac{mm}{N}$

$\delta_f = \left(1.141 \cdot 10^{-6}\right) \frac{mm}{N}$

f)

$\rho := \operatorname{atan}\left(\frac{\mu}{\cos\left(\frac{\alpha}{2}\right)}\right) = 10.467\ °$ $\varphi := \operatorname{atan}\left(\frac{p}{\pi \cdot d_2}\right) = 3.167\ °$

$\alpha = 60\ °$ $\mu = 0.16$ $p = 1.25\ mm$ $d_2 = 7.19\ mm$

$D_{Km} := \frac{d_w + d_h}{2} = 11\ mm$

$d_w = 13\ mm$ $d_h = 9\ mm$

$M_G := F_{Mmin} \cdot \frac{d_2}{2} \cdot \tan(\varphi + \rho) = 7.455\ N \cdot m$

$F_{Mmin} = 8.549\ kN$ $d_2 = 7.19\ mm$ $\varphi = 0.055$ $\rho = 0.183$

$M_{SK} := F_{Mmin} \cdot \frac{D_{Km}}{2} \cdot \mu = 7.523\ N \cdot m$

$F_{Mmin} = 8.549\ kN$ $D_{Km} = 11\ mm$ $\mu = 0.16$

$M_{Amin} := M_G + M_{SK} = 14.978\ N \cdot m$

g)

$F_{SA} := \phi \cdot F_A = 1.663\ kN$ $d_s := \frac{d_2 + d_3}{2} = 6.83\ mm$

$\phi = 0.216$ $d_2 = 7.19\ mm$

$F_A = 7.697\ kN$ $d_3 = 6.47\ mm$

$\sigma_z := \frac{F_{Mmax} + F_{SA}}{\frac{\pi}{4} \cdot (d_s)^2} = 418.734\ MPa$

$F_{Mmax} = 13.678\ kN$ $F_{SA} = 1.663\ kN$ $d_s = 6.83\ mm$

$W_p := \frac{\pi}{16} \cdot d_s^{\ 3} = 62.559\ mm^3$

$M_G := F_{Mmax} \cdot \frac{d_2}{2} \cdot \tan(\varphi + \rho) = 11.928\ N \cdot m$

$F_{Mmax} = 13.678\ kN$ $\qquad \varphi = 0.055$ $\qquad \rho = 0.183$

$\tau := \frac{M_G}{W_p} = 190.67\ MPa$ $\qquad M_G = 11.928\ N \cdot m$

$\sigma_V := \sqrt{\sigma_z^{\ 2} + 3 \cdot \tau^2} = 533.295\ MPa$

$\sigma_z = 418.734\ MPa$ $\qquad \tau = 190.67\ MPa$

$j := \left(\frac{\sigma_V}{R_{0.2}}\right)^{-1} = 1.688$

$\sigma_V = 533.295\ MPa$ $\qquad R_{0.2} = 900\ MPa$

$\sigma_{dy} := \frac{\phi \cdot F_A}{\frac{\pi}{4} \cdot (d_s)^2} = 45.394\ MPa$ $\qquad \phi = 0.216$ $\qquad d_s = 6.83\ mm$ $\qquad F_A = 7.697\ kN$

$\sigma_{dy} < 110\ MPa = 1$

$j_{dyn} := \left(\frac{\sigma_{dy}}{110\ MPa}\right)^{-1} = 2.423$ $\qquad j_{dyn} > 1.8 = 1$

$F_K := F_{Mmin} - (1 - \phi) \cdot F_A = 2.515\ kN$ $\qquad F_{Mmax} = 13.678\ kN$

$\phi = 0.216$

$F_K > \frac{F_{Kges}}{i} = 1$ $\qquad F_A = 7.697\ kN$

$$p := \frac{F_{Mmax} + F_{SA}}{\frac{\pi}{4} \cdot \left(d_w^{\ 2} - d_h^{\ 2}\right)} = 221.971\ MPa$$

$F_{Mmax} = 13.678\ kN$

$F_{SA} = 1.663\ kN$

$d_w = 13\ mm$ $d_h = 9\ mm$

$p_{zulf} := 800\ MPa$

$p < p_{zulf} = 1$